# Miami University Technical Report: Dynamic Resource Allocation in Integrated Optical Wireless Access Architectures (Version 1, October 2008)

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**Abstract:** We consider integrated optical-wireless access network architectures. We propose and evaluate dynamic resource allocation algorithms for such architectures with an explicit incorporation of the reconfiguration delays. Algorithmic complexity results are also presented.

# 1. Introduction and Motivation

Integration of optical and wireless technologies at the network edge promises a combination of the key advantages of the two: high bandwidth and long reach due to optics, and easy deployment and mobility due to wireless. The advances in key technologies such as WiMax, Passive Optical Networks (PON), and Gigabit Ethernet have lead to many network architecture proposals for integrated optical-wireless access, including a combination of WiMax with PON, Gigabit Ethernet PON (GEPON), or optical fiber [1, 2, 3]; and UWB with WDM/TDM-PON [4]. One of the key advantages of such integrated architectures is seen as the facilitation of the dynamic allocation of access related functionalities between the wireless and optical components [1]. However, more research needs to be done on resource allocation problems in integrated access architectures, such as [2], which addresses the antennae assignment aspect of the integrated PON-WiMax MIMO systems. Wireless and optical networks have distinct characteristics which deserve specific attention for integrated access systems, since it is envisioned that these systems would support service-specific resource allocation in both the optical tier of the access network and across the wireless interface [1].

In this paper, we consider a dynamic resource allocation problem in integrated opticalwireless networks which takes into account two distinct aspects of the two technologies: the time-varying nature of the wireless medium, and the set-up delay associated with reconfiguring optical networks. Dynamic reconfiguration in optical networks has been considered in various contexts, including optical metro access networks in [5], where reconfiguration delay was recently modeled as a loss of service with a probabilistic distribution. In [5], these reconfigurations were mainly triggered by traffic variability. In this paper, we consider the reconfigurations to result from not only traffic changes, but also the time-varying nature of the wireless channels. In the presence of multiple users sharing a wireless medium, each user may experience a different channel quality on each channel. This leads to the possibility of switching the users assigned to the channels as well as changing the transmission rates based on the time-varying *channel state*. Opportunistic scheduling and rate-adaptation mechanisms exploit the variations in the channel states experienced by different users through the allocation of resources to users with better channel quality. However, in the presence of an integrated optical-wireless architecture, where each flow receives a consistent bandwidth allocation in both the optical and the wireless domains as in the architectures of [3, 1], this end-to-end rate adaptation and resource reassignment may lead to a reconfiguration delay. We model this reconfiguration delay as a loss of service for a fixed duration  $\tau$ , and solve the dynamic resource allocation problem for throughput maximization. Our basic framework is flexible to accommodate various architectures, including, for example, the two architectures presented in [1], where wireless frequency subbands of a WiMax base station may be dynamically allocated among its subscribers in conjunction with the optical tier of the access network.

### 2. Dynamic Resource Allocation for Integrated Optical-Wireless Access

We assume that the wireless spectrum is divided into *F* orthogonal channels, and is shared by *N* flows served by a common base station. Each flow may transmit or receive on multiple channels. Time is slotted. The transmission rate for flow *n* in time slot *i* on channel *f* may take one of *M* discrete values, but is upper bounded by the corresponding capacity  $c_{n,f}(i)$ , which represents a combination of the channel state and the traffic pattern. We assume that at the beginning of time slot *i*, knowledge of  $c_i, \ldots, c_{i+K}$  is available, where *K* represents the prediction horizon. The channel assignment and transmission rates can be dynamically modified in order to improve the system performance with a reconfiguration penalty of  $\tau$  time slots on that channel. The goal is to maximize the total throughput  $\sum_{n=1}^{N} \sum_{f=1}^{F} \sum_{t=1}^{T} x_{n,f}(t)$ , where *T* is the overall transmission period, and  $x_{n,f}(t)$  is the selected transmission rate for flow *n* in time slot *t* on channel *f*.

#### 3. Theoretical Results on Off-line Version of the Problem

**Theorem:** Consider the offline version of the problem, i.e., the case with  $K \ge T$ . We present the following summary of results.

**1.** Given a target throughput and an instance of the off-line problem, determining whether the throughput is achievable or not is NP-hard.

- **2.** There exists a  $\tau$ -approximation to the offline problem.
- **3.** The single-flow version of the problem can be solved using dynamic programming.

# In the following, we discuss the results above in more detail.

# 3.1. Complexity Analysis

Consider the decision version of the off-line problem, where we are given an instance of the problem as well as a target throughput value, and our goal is to determine whether the given problem instance has a solution that would reach or exceed the target throughput value.

**Lemma 1**: When  $\tau = 1$ , the off-line problem is equivalent to the predictive multi-user multichannel scheduling problem considered in [6].

The equivalence of the two problems can be seen by replacing the end-to-end transmission capacity variables in the current problem when  $\tau = 1$  with the link capacity variables in [6]. We refer the reader to [6] for details.

**Lemma 2**: The decision version of the predictive multi-user multi-channel scheduling problem considered in [6] is NP-complete. (See [7] for the proof.)

**Theorem:** The decision version of the off-line integrated scheduling problem is NPcomplete.

**Proof:** From Lemmas 1 and 2, the special case of the problem with  $\tau = 1$  is NP-complete. It follows that the general problem with an arbitrary  $\tau$  is also NP-complete.

#### 3.2. An Off-line Approximation Algorithm: OTSS

Having established the complexity of the off-line problem, we next describe an approximation algorithm, namely the Off-line Time Slot Selection (OTSS) algorithm. As we discuss in more detail below, OTSS is a  $\tau$ -Approximation, and has a low complexity. We assume that each flow may use at most one channel throughout this section, although the algorithm can be easily modified to accommodate more channels per user.

**Off-line Time Slot Selection Algorithm:** OTSS consists of two phases. In phase I, for each time-slot, the best channel and rate assignment is calculated, completely ignoring the reconfiguration penalty constraints. In particular, for each time slot, a bipartite graph is formed, where the nodes consist of the set of flows, and the set of channels. The weight of a link between a flow node and a channel node represents the capacity available for that flow on that channel. OTSS then runs a maximum weighted matching algorithm to assign channels to flows while maximizing the total throughput for that time-slot. We note that maximum weighted matching for a bipartite graph can be carried out using a simple linear program, and has low complexity [8]. Note that this process is repeated for each time-slot. Let  $\kappa_i$  denote the value of the maximum weighted matching for time-slot *i*.

Note that the assignment obtained as a result of phase I is not necessarily feasible, since it does not consider the reconfiguration overhead. Phase II of OTSS optimally selects a subset of the time-slots for the actual transmission, so that reconfiguration constraint is not violated. The Mixed Integer Linear Program (MILP) in Figure 1 accomplishes this. The binary decision variable  $I_i$  determines whether time-slot *i* is actually used for transmission or not. If  $I_i = 1$ , then the previous  $\tau$  transmissions should all be zero, as accomplished through constraint 1, which becomes  $\sum_{j=i-\tau}^{i-1} I_j \leq 0$ . If  $I_i = 0$ , then constraint 1 becomes  $\sum_{j=i-\tau}^{i-1} I_j \leq \tau$ , which is automatically satisfied.

OTSS Phase II MILP Binary Decision variables:  $I_i$ Maximize:  $\sum_{i=1}^{T} \kappa_i I_i$ s.t. Each time slot is either zero, or is preceded by  $\tau$  zero time slots For  $\forall i, 1 \le i \le T$ 1.  $\sum_{j=i-\tau, j>0}^{i-1} I_j \le \tau(1-I_i)$ 

# Fig. 1. OTSS Phase II.

**Performance Bound for OTSS:** We next show that the throughput achieved by OTSS is at least  $(1/\tau)\sum_{i=1}^{T} \kappa_i$ , and that OTSS is a  $\tau$ -approximation algorithm.

**Lemma:** OTSS results in a throughput OTSS(T) that is at least  $\sum_{i=1}^{T} \kappa_i / \tau$ .

*Proof:* OTSS phase II optimally selects the time-slots to be used for transmission without violating the reconfiguration penalty constraints. Therefore, its throughput should be at least as high as any other feasible time-slot selection solution. Consider the following family of algorithms, each of which results in a feasible solution. Divide the time-slots t = 1 to T into  $\tau$  equivalence classes, where equivalence class 1 consists of time-slots with  $t = 1 \pmod{\tau}$ , equivalence class 2 consists of time-slots with  $t = 2 \pmod{\tau}$ , etc. Clearly, an algorithm that allows only one equivalence class to be used for transmission would yield a legitimate solution.

$$OTSS(T) \geq \sum_{t=0(mod\tau)} \kappa_t,$$
  

$$OTSS(T) \geq \sum_{t=1(mod\tau)} \kappa_t,$$
  

$$\dots OTSS(T) \geq \sum_{t=\tau-1(mod\tau)} \kappa_t$$
  

$$\Rightarrow \tau OTSS(T) \geq \sum_{t=1}^T \kappa_t$$

**Theorem:** OTSS is a  $\tau$  approximation to the off-line problem.

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*Proof:* Assume that the optimal throughput achievable by any off-line algorithm is OFO. Consider also the throughput of the optimal solution when the reconfiguration penalty constraint is relaxed, say  $OFO_{rel}$ . Clearly,  $OFO_{rel} = \sum_{t=1}^{T} \kappa_t$ . Since  $OFO \leq OFO_{rel}$ , and  $\tau OTSS(T) \geq \sum_{t=1}^{T} \kappa_t$  from the Lemma above, the result follows.

# 4. Dynamic Integrated Resource Allocation Algorithm

Dynamic Integrated Resource Allocation (DIRA), an online scheduling algorithm, creates tentative K + 1 time-slot schedules for each flow and channel by running a set of mixed integer linear programs (MILPs) in parallel. The DIRA MILP for flow *n* and channel *f* is shown in Figure 2. The objective function *Z* is a weighted sum of the transmission rates, where the weights  $0 \le w_j \le 1$  are introduced as algorithmic parameters that determine whether the algorithm has a short-term or long-term focus. DIRA collects all the MILP solutions, and allocates each channel to the flow that has the highest Z, without exceeding the maximum allowed number of channels per flow. We note that new information becomes available after each time slot, and the procedure is repeated. Therefore, new tentative schedules are formed starting from the next time slot.

#### 5. Performance Evaluation

We illustrate the operation of DIRA in a two-flow single-channel system with reconfiguration penalty  $\tau = 2$ , and K = 3. At each time-slot, a 4-time slot tentative schedule is created for each flow. The capacity values are created randomly as integers from 1 to 12. We use two different weight sets  $\{w_i, w_{i+1}, w_{i+2}, w_{i+3}\}$  for the objective function: WS1=  $\{1, 1, 0.5, 0.5\}$ , and WS2= $\{1, 0, 0, 0\}$ . WS1 weighs short-term transmissions moderately higher, whereas WS2 has an extreme short-term focus by weighing only the current slot. Figure 2 shows the DIRA schedules for both flows, with total WS1 and WS2 throughputs of 75 and 66, respectively. As expected, incorporating longer term transmission opportunities in the current decision improves performance. Interestingly, we observed that having a moderate short-term focus typically resulted in higher throughput than weighing all slots equally (not shown). We believe that this is due to the sliding window nature of the algorithms: as new information becomes available, previous tentative schedules may become obsolete.

#### 6. Conclusion

Optical-wireless integration at the network edge calls for a joint approach to dynamic allocation. We considered a dynamic resource allocation problem in integrated optical-wireless architectures addressing the reconfiguration delay and time-varying channel quality. We presented theoretical results, and an online throughput optimization algorithm. The results suggest

Then,

DIRA: MILP for flow n on channel f

**Goal:** Creates a (K+1)-slot schedule (from slot *i* to *i* + *K*) for user *n* on channel *f*  **Given:** (i) lnz(f): the last time-slot with a non-zero transmission on channel *f* by any user (ii)  $x_{n,f}(i-\tau), \ldots, x_{n,f}(i-1)$ : the previous  $\tau$  transmission rates for flow *n* on channel *f*, (iii)  $c_{n,f}(i), \ldots c_{n,f}(i+K)$ : the capacity for the current and next *K* time slots **Decision Variables:**  $x_{n,f}(i), \ldots x_{n,f}(i+K)$ : tentative transmission rates Auxiliary Binary Variables:  $\alpha_j, \beta_j$ , for  $i \le j \le i+K$  **Maximize**  $Z = \sum_{j=i}^{i+K} w_j x_{n,f}(j)$  // Maximize weighted throughput **Subject to:** Constraint Set 1: Capacity constraints:  $x_{n,f}(j) \le c_{n,f}(j)$ , for  $i \le j \le i+K$ Constraint Set 2: // Impose penalty due to earlier transmissions if needed If  $lnz(f) > (i-\tau)$  AND  $x_{n,f}(i-1) = 0$ Set:  $x_{n,f}(j) = 0$ , for  $i \le j \le lnz(f) + \tau$ Constraint Set 3: // Rate Continuity and Penalty Constraints For all *j* s.t.  $max(i, lnz(f) + \tau + 1) \le j \le i + \tau$ (1)  $x_{n,f}(j) \le Y(1 - \beta_j) + 2Y\alpha_j$ , (2)  $x_{n,f}(j) \le x_{n,f}(j - 1) + 2Y(\alpha_j) + Y\beta_j$ , (3)  $x_{n,f}(j) \ge x_{n,f}(j - 1) - 2Y(\alpha_j) - Y\beta_j$ , (4)  $\sum_{k=j-\tau}^{j-1} \le Y\tau(1 - \alpha_j)$ 

Fig. 2. Dynamic Integrated Resource Allocation: Decision MILP for flow n on channel f.

Time	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Cap for Flow 1	2	9	7	8	3	11	5	3	4	9	4	5	2	1	6	9	3	2	12	5
Cap for Flow 2	6	2	11	8	12	4	3	8	2	6	5	9	9	1	1	10	12	1	2	9
WS1: Flow 1 Rate	0	7	7	7	0	0	5	0	0	0	0	0	0	0	0	0	0	0	12	0
WS1: Flow 2 Rate	0	0	0	0	0	0	0	0	0	5	5	5	5	0	0	10	10	0	0	9
WS2: Flow 1 Rate	0	0	0	8	0	0	5	0	0	9	0	0	0	0	0	0	0	0	0	0
WS2: Flow 2 Rate	6	0	0	0	0	0	0	0	0	0	0	0	9	0	0	10	10	0	0	9

Fig. 3. DIRA transmission schedule with reconfiguration penalty  $\tau = 2$ , and prediction horizon K = 3 for a two-flow single-channel system with two weight parameter sets.

that algorithms with a moderately short-term focus yield better throughput than the other two extremes. We also observed that increasing the number of flows increases the throughput due to multi-user diversity (not shown.) We plan to incorporate fairness constraints in our future work.

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